

MIDTERM EXAMINATION I

Directions: Do all three problems, which have unequal weight. This is a closed-book closed-note exam except for one $8\frac{1}{2} \times 11$ inch sheet containing any information you wish on both sides. Calculators are not needed, but you may use one if you wish. Use a bluebook. Do not use scratch paper – otherwise you risk losing part credit. Cross out rather than erase any work that you wish the grader to ignore. Justify what you do. Express your answer in terms of the quantities specified in the problem. Box or circle your answer.

Problem 1. (30 points)

You are fishing on a glassy calm, essentially flat lake at the Earth's equator on March 21 (when the sun rises at 6 AM, is directly overhead at noon, and sets at 6 PM). Wearing Polaroid sunglasses, you can see the sun, but you can't see its reflection in the water ($\mu = \mu_0$, refractive index $n = \frac{4}{3}$).

(a) (10 points) Is the transmission axis of your Polaroid sunglasses vertical or horizontal? Why?

(b) (20 points) What time of morning is it? (If you aren't using a calculator, you may leave your answer in the form of an expression involving an arc trigonometric function.)

Problem 2. (35 points)

A plane wave of wavelength λ propagates in the z direction toward a baffle in the plane $z = 0$ made of opaque material. In the baffle, four pinholes of negligible diameter are drilled so that each hole lies on a different corner of a square with side d (the holes are at $|x| = |y| = d/2$). The resulting diffraction pattern is observed on a screen at $z = Z \gg d$, also with $Z \gg d^2/\lambda$ so that Fraunhofer conditions apply. The diffraction pattern is characterized by the variables $\psi_x \equiv X/Z$ and $\psi_y \equiv Y/Z$, where X and Y are the screen coordinates along \hat{x} and \hat{y} .

Provide the irradiance ratio

$$\frac{I(\psi_x, \psi_y)}{I(0, 0)}$$

for the following four different sets of conditions, and justify your answers:

(a) (7 points) Three of the four pinholes are blocked.

(b) (7 points) Only the two pinholes at negative y are blocked.

(c) (7 points) No pinholes are blocked, but a right-hand circular polarizer is placed in front of the two pinholes at positive y , and a left-hand circular polarizer is placed in front of the two pinholes at negative y .

(d) (14 points) No pinholes are blocked, and no polarizers are in use.

[Hint: Consider the possibility that not all of the correct answers are nontrivial, and the possibility that the correct answers are not all different from each other.]

Problem 3. (35 points) An electron at rest, when subjected to a weak electromagnetic field at angular frequency ω , emits electromagnetic (EM) radiation at that same frequency. Now consider a free-electron laser, in which an electron moves at constant relativistic speed βc along the axis z of a structure consisting of a series of identical magnets, each separated from the next by the same $\Delta z = \ell$. These magnets produce a static EM field in the lab which is felt as a periodic EM field in the electron's rest frame.

(a) (15 points) What angular frequency ω' of the periodic EM field is experienced by the electron in its own rest frame?

(b) (20 points) In the lab frame, what is the wavelength λ of the EM radiation that is emitted by the electron in the forward direction? [Hint: your answer should depend only on β and ℓ .]